Magic of Fluctuations:

From Fluctuations in Quantum Information to Magic

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November 2024

Random Quantum Circuits:

Article

Quantum supremacy using a programmable superconducting processor

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The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor[1]. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits[2–7] to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 2^{53} (about 10^{16}). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years. This dramatic increase in speed compared to all known classical algorithms is an experimental realization of quantum supremacy[8–14] for this specific computational task, heralding a much-anticipated computing paradigm.

Article

Phase transitions in random circuit sampling

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Undesired coupling to the surrounding environment destroys long-range correlations in quantum processors and hinders coherent evolution in the nominally available computational space. This noise is an outstanding challenge when leveraging the computation power of near-term quantum processors[1]. It has been shown that benchmarking random circuit sampling with cross-entropy benchmarking can provide an estimate of the effective size of the Hilbert space coherently available[2–8]. Nevertheless, quantum algorithms' outputs can be trivialized by noise, making them susceptible to classical computation spoofing. Here, by implementing an algorithm for random circuit sampling, we demonstrate experimentally that two phase transitions are observable with cross-entropy benchmarking, which we explain theoretically with a statistical model. The first is a dynamical transition as a function of the number of cycles and is the continuation of the anti-concentration point in the noiseless case. The second is a quantum phase transition controlled by the error per cycle; to identify it analytically and experimentally, we create a weak-link model, which allows us to vary the strength of the noise versus coherent evolution. Furthermore, by presenting a random circuit sampling experiment in the weak-noise phase with 67 qubits at 32 cycles, we demonstrate that the computational cost of our experiment is beyond the capabilities of existing classical supercomputers. Our experimental and theoretical work establishes the existence of transitions to a stable, computationally complex phase that is reachable with current quantum processors.

Nature, 574,505–510 **(2019)**

Nature, 634, 328–333 **(2024)**

Random Quantum Circuits:

RESEARCH

QUANTUM SIMULATION

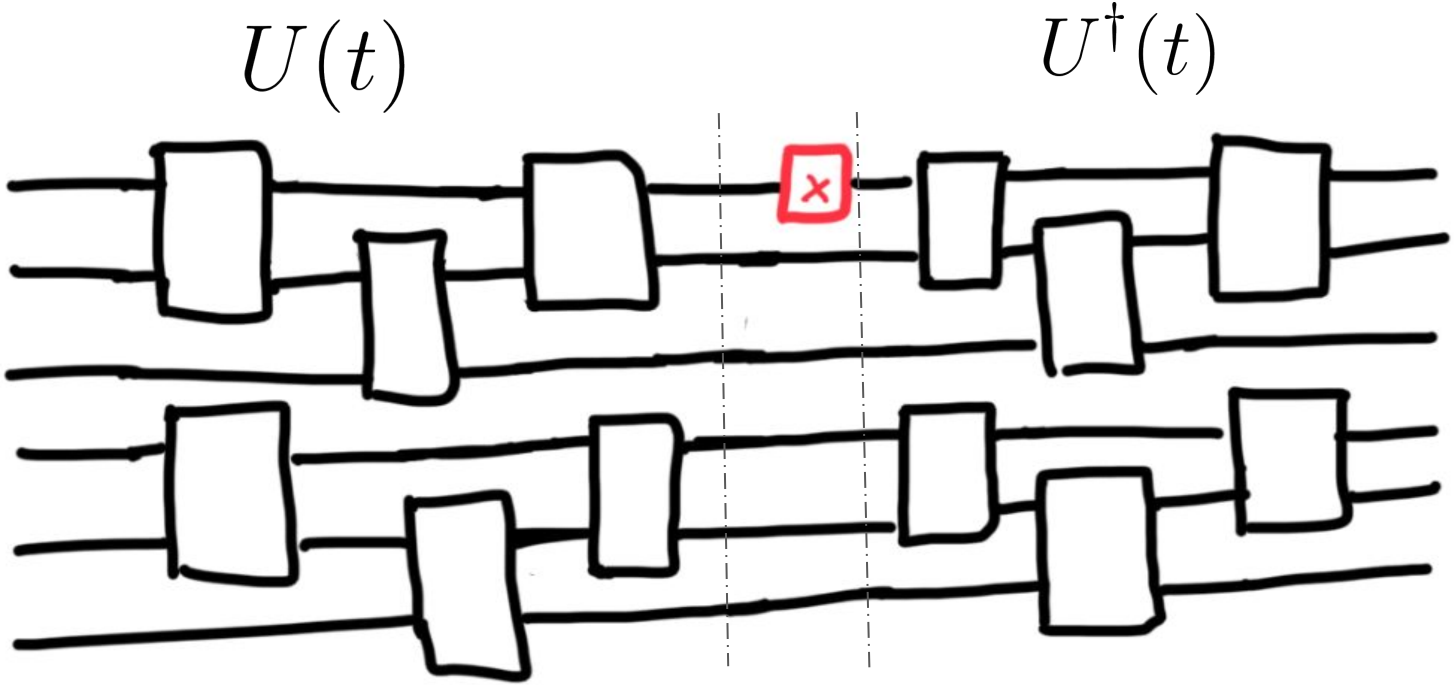
Information scrambling in quantum circuits

Xiao Mi[1]†, Pedram Roushan[1]†, Chris Quintana[1]†, Salvatore Mandrà[2,3], Jeffrey Marshall[2,4], Charles Neill[1], Frank Arute[1], Kunal Arya[1], Juan Atalaya[1], Ryan Babbush[1], Joseph C. Bardin[1,5], Rami Barends[1], Joao Basso[1], Andreas Bengtsson[1], Sergio Boixo[1], Alexandre Bourassa[1,6], Michael Broughton[1], Bob B. Buckley[1], David A. Buell[1], Brian Burkett[1], Nicholas Bushnell[1], Zijun Chen[1], Benjamin Chiaro[1], Roberto Collins[1], William Courtney[1], Sean Demura[1], Alan R. Derk[1], Andrew Dunsworth[1], Daniel Eppens[1], Catherine Erickson[1], Edward Farhi[1], Austin G. Fowler[1], Brooks Foxen[1], Craig Gidney[1], Marissa Giustina[1], Jonathan A. Gross[1], Matthew P. Harrigan[1], Sean D. Harrington[1], Jeremy Hilton[1], Alan Ho[1], Sabrina Hong[1], Trent Huang[1], William J. Huggins[1], L. B. Ioffe[1], Sergei V. Isakov[1], Evan Jeffrey[1], Zhang Jiang[1], Cody Jones[1], Dvir Kafri[1], Julian Kelly[1], Seon Kim[1], Alexei Kitaev[1,7], Paul V. Klimov[1], Alexander N. Korotkov[1,8], Fedor Kostritsa[1], David Landhuis[1], Pavel Laptev[1], Erik Lucero[1], Orion Martin[1], Jarrod R. McClean[1], Trevor McCourt[1], Matt McEwen[1,9], Anthony Megrant[1], Kevin C. Miao[1], Masoud Mohseni[1], Shirin Montazeri[1], Wojciech Mruczkiewicz[1], Josh Mutus[1], Ofer Naaman[1], Matthew Neeley[1], Michael Newman[1], Murphy Yuezhen Niu[1], Thomas E. O'Brien[1], Alex Opremcak[1], Eric Ostby[1], Balint Pato[1], Andre Petukhov[1], Nicholas Redd[1], Nicholas C. Rubin[1], Daniel Sank[1], Kevin J. Satzinger[1], Vladimir Shvarts[1], Doug Strain[1], Marco Szalay[1], Matthew D. Trevithick[1], Benjamin Villalonga[1], Theodore White[1], Z. Jamie Yao[1], Ping Yeh[1], Adam Zalcman[1], Hartmut Neven[1], Igor Aleiner[1], Kostyantyn Kechedzhi[1]*, Vadim Smelyanskiy[1]*, Yu Chen[1]*

Interactions in quantum systems can spread initially localized quantum information into the exponentially many degrees of freedom of the entire system. Understanding this process, known as quantum scrambling, is key to resolving several open questions in physics. Here, by measuring the time-dependent evolution and fluctuation of out-of-time-order correlators, we experimentally investigate the dynamics of quantum scrambling on a 53-qubit quantum processor. We engineer quantum circuits that distinguish operator spreading and operator entanglement and experimentally observe their respective signatures. We show that whereas operator spreading is captured by an efficient classical model, operator entanglement in idealized circuits requires exponentially scaled computational resources to simulate. These results open the path to studying complex and practically relevant physical observables with near-term quantum processors.

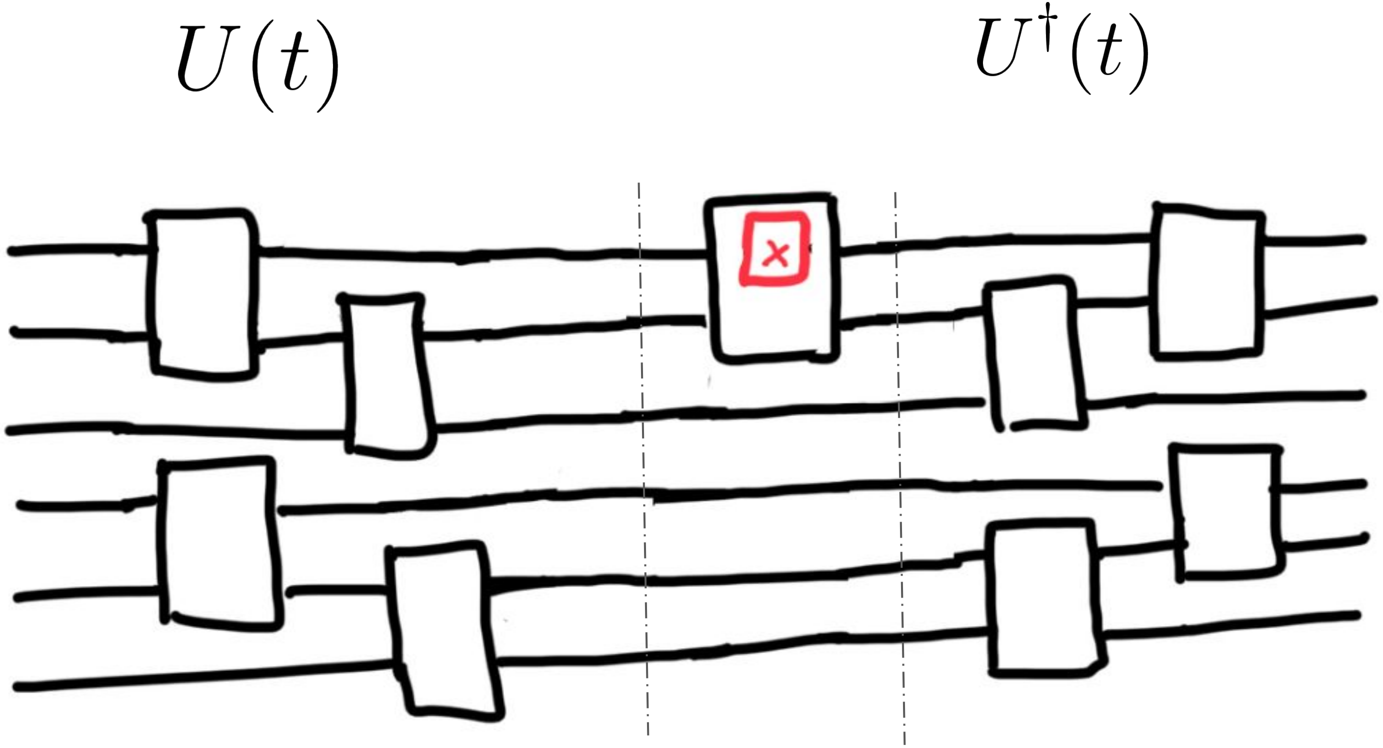
Science 374, 1479–1483 **(2021)**

Information Scrambling

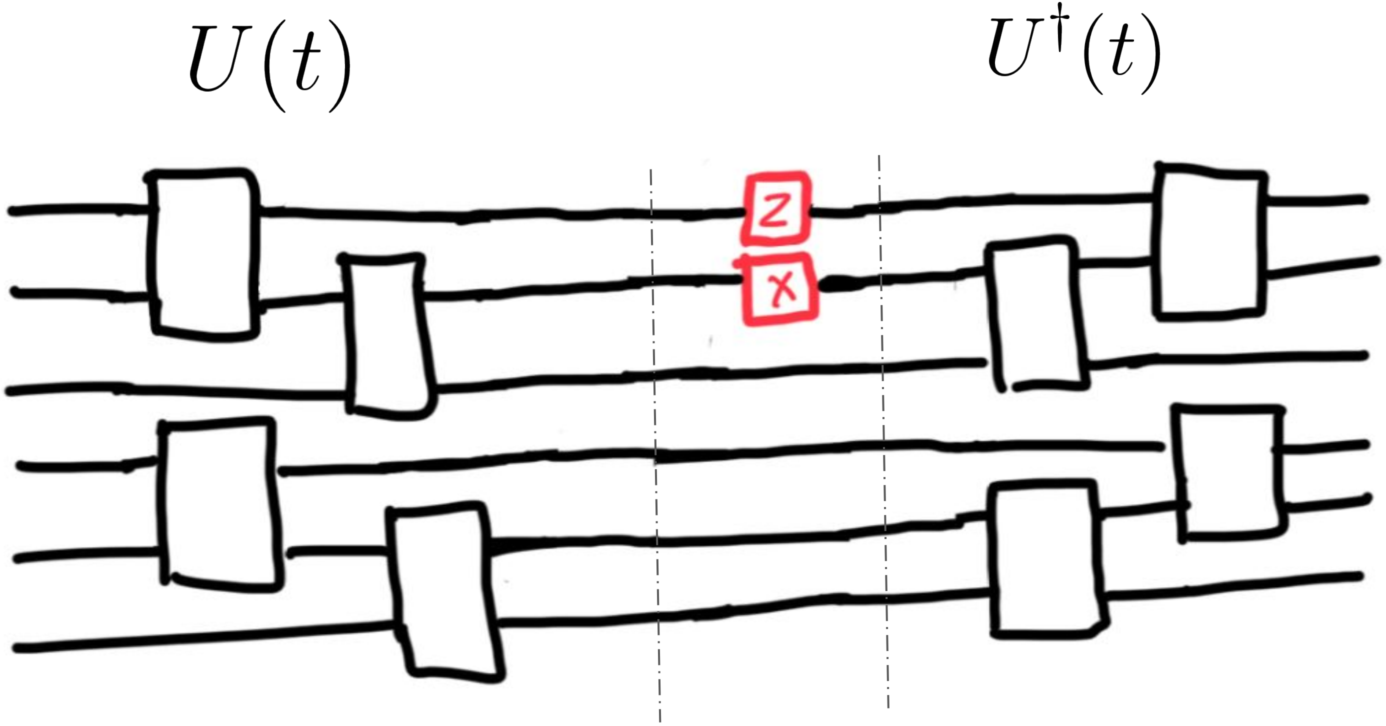


$$\rho(0) = X_1$$

Information Scrambling

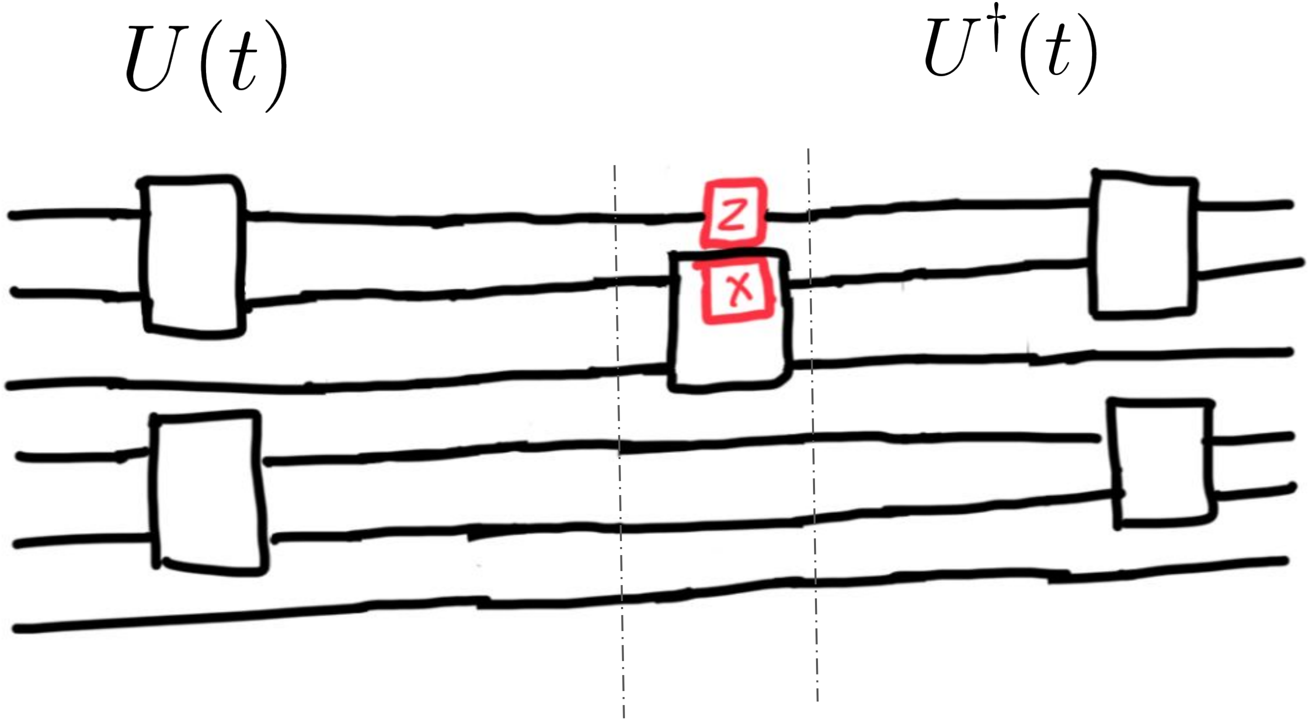


Information Scrambling

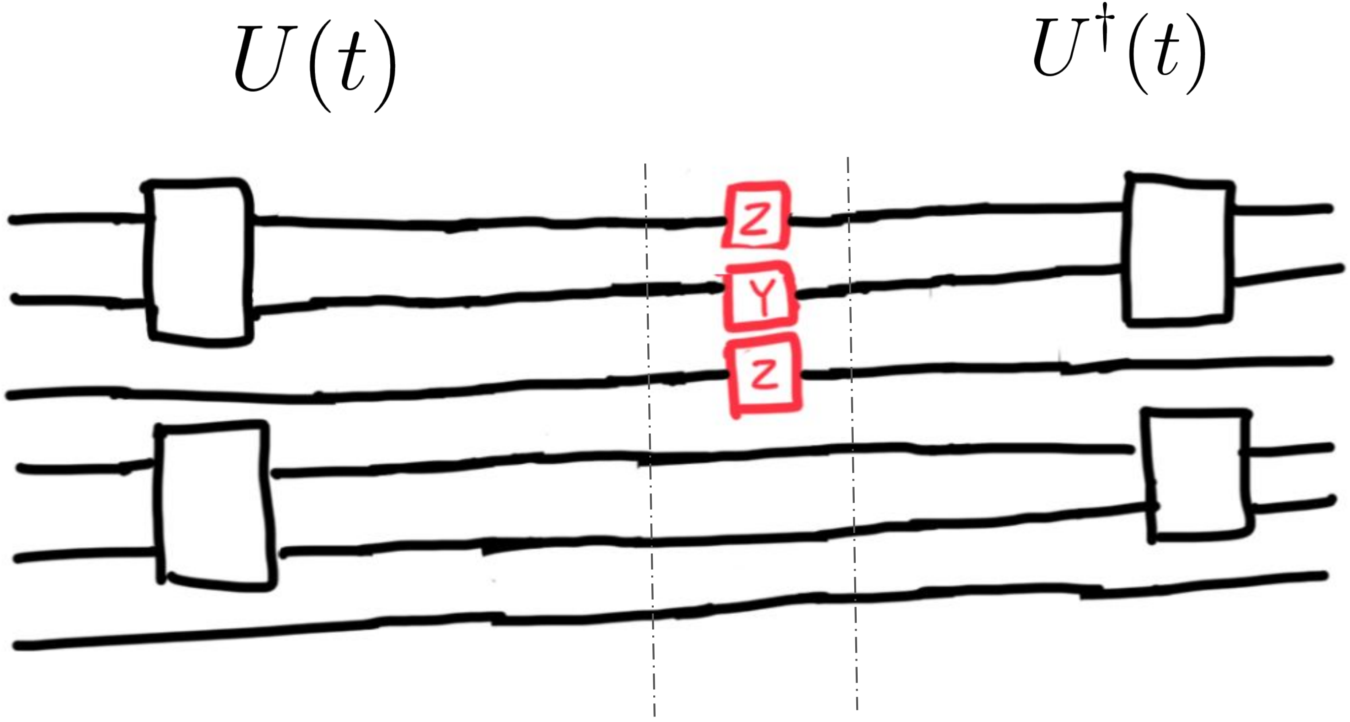


$$\rho(\delta t) = c_1 Z_1 + c_2 X_2$$

Information Scrambling



Information Scrambling

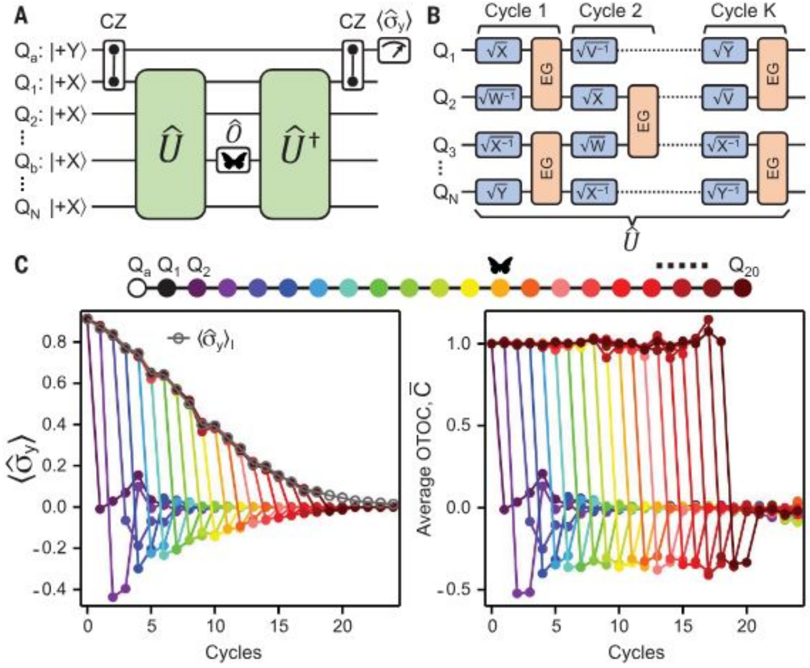


Information Scrambling

$$\rho(0) = X_1 \quad \Longrightarrow \quad \rho(t) = U(t)\rho(0)U^\dagger(t) = \sum c_i P_i$$

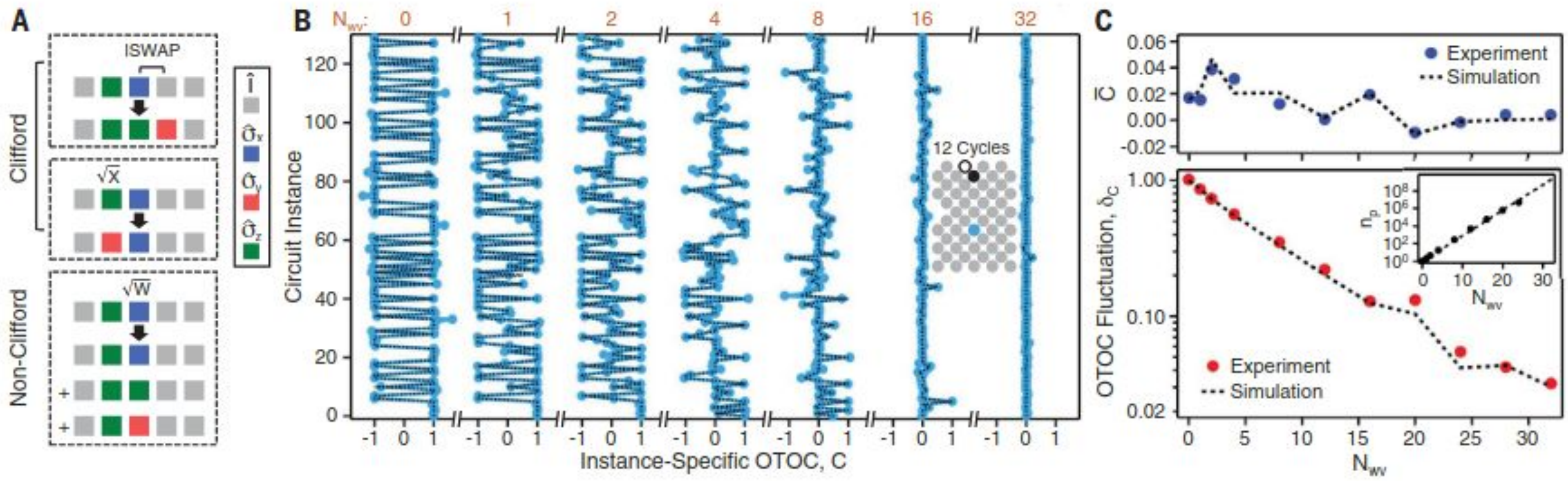
$$\text{OTOC}(t) = \text{Re}(\langle A^\dagger(t) B^\dagger A(t) B \rangle),$$

Information scrambling in quantum circuits



Science 374, 1479–1483 **(2021)**

Information scrambling in quantum circuits



Science 374, 1479–1483 **(2021)**

Explanation:

Definition 3

The OTOC magic of an *n*-qubit unitary *U* is

$$O_M(U) \equiv \max_{P_a, P_b \in \mathscr{P}_2^{\otimes n}} [1 - |\mathrm{OTOC}(U)|],$$

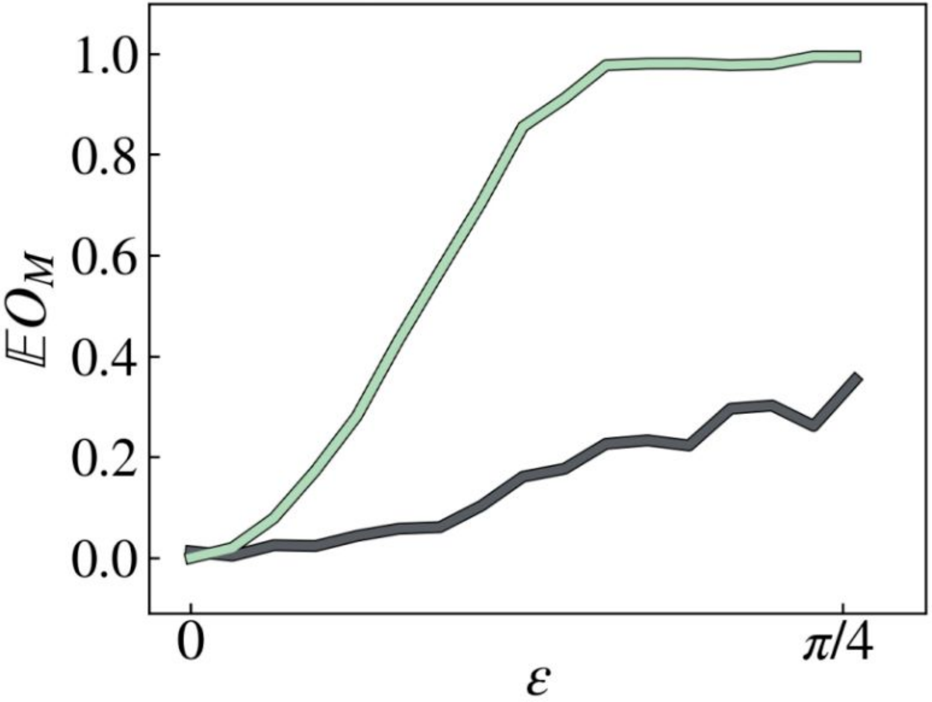
Theorem 1

If $\mathop{\mathbb{E}}_{U \sim \mathscr{E}} \mathrm{OTOC}(U) \to 0$, then

$$\mathop{\mathbb{E}}_{U \sim \mathscr{E}} O_M(U) \geq 1 - \delta.$$

Proc. Natl. Acad. Sci. U.S.A. 120 (17) e2217031120 **(2023)**

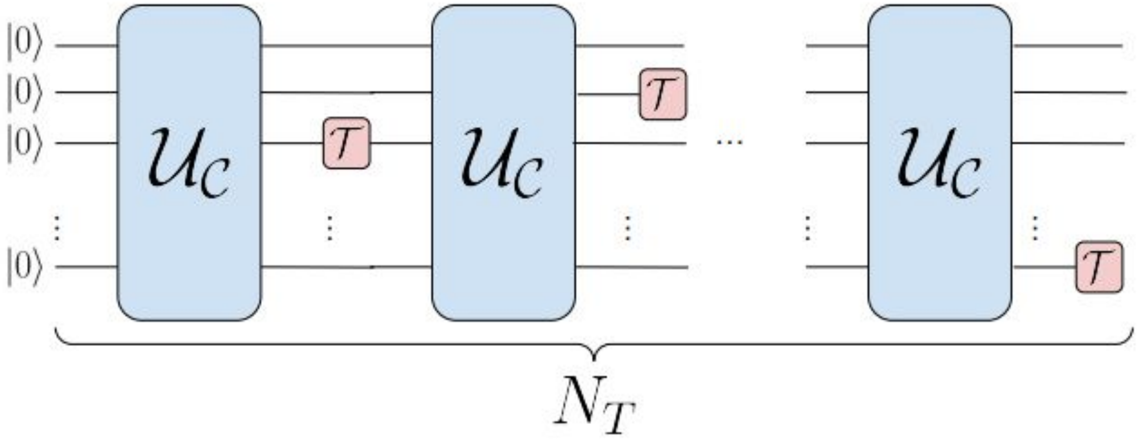
Explanation:



Plot of $\underset{U\sim\mathcal{E}}{\mathbb{E}} O_M(U)$ (green) and $1-\delta$ (black) against ε. These quantities are empirically computed using 50 random samples from the ensemble $\mathcal{E}$ generated by Eq. 11.

Proc. Natl. Acad. Sci. U.S.A. 120 (17) e2217031120 **(2023)**

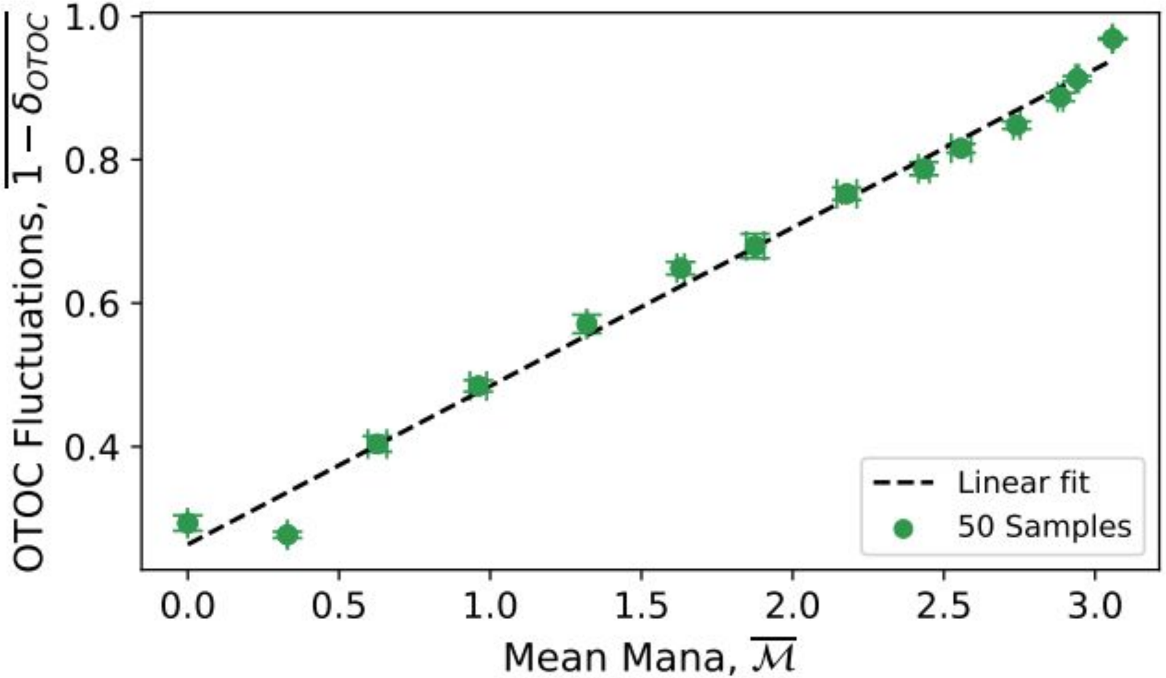
There must be a better explanation:



For Qutrit systems

A.A. & E.G. *SciPost Phys.* 16, 043 **(2024)**

Mana:



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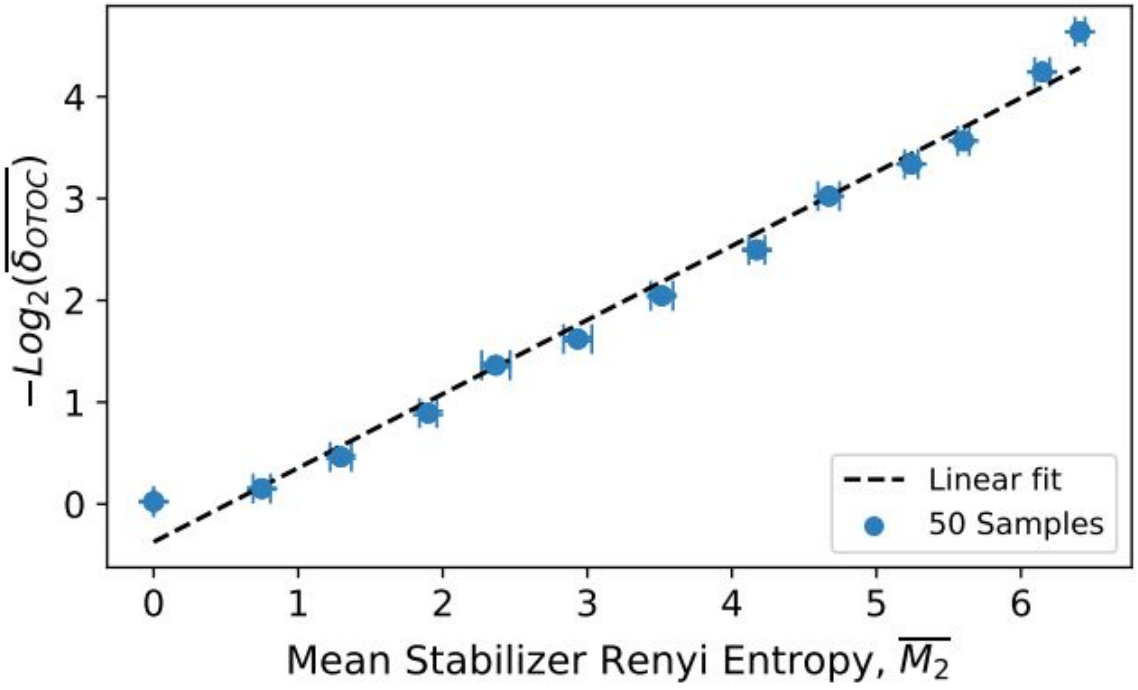
Stabilizer Renyi Entropy:



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Information scrambling quantifiers:

Correlators-based

Entropic-based

Entropic-based quantifiers:

- Entanglement entropy
- Mutual information
- Topological entanglement entropy
- Tripartite information
- …

Mutual information:

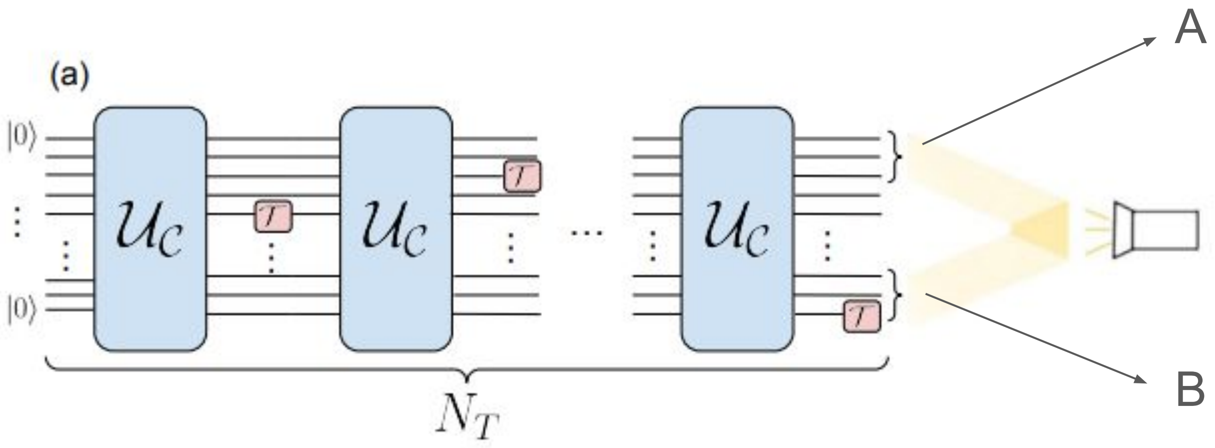
By dividing the whole system into three subsystems of A, B and the complement of A and B:

$$I^{(2)} := S_A^{(2)} + S_B^{(2)} - S_{AB}^{(2)}$$

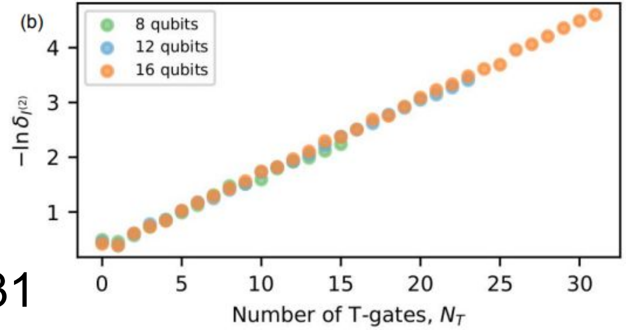
$$S_X^{(2)} \equiv -\log_2 tr\rho_X^2$$

A.A., J.H.,C.K. & E.G., *arXiv:*2408.03831

Mutual information meets magic:

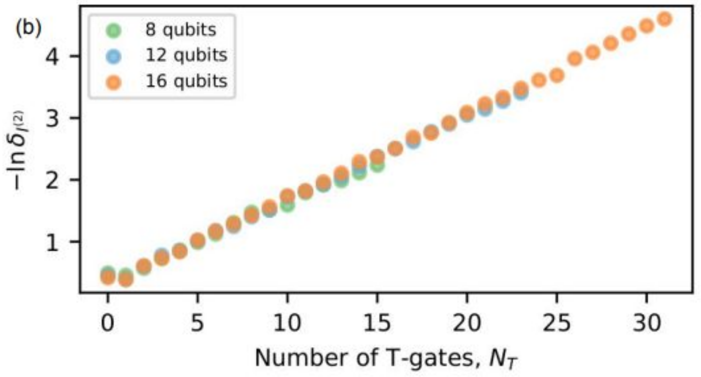


When |A|=|B|=N/4 ⇒



A.A., J.H.,C.K. & E.G., *arXiv:*2408.03831

Mutual information meets magic:



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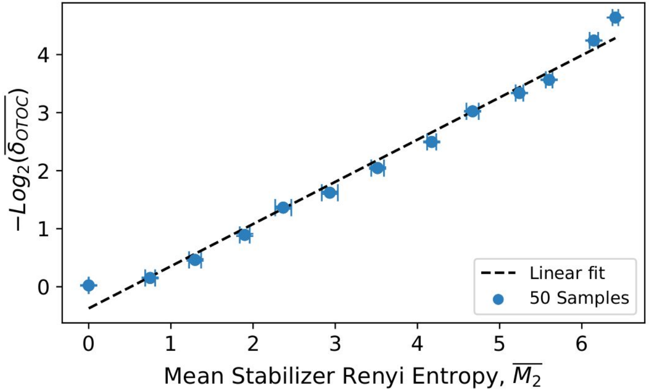
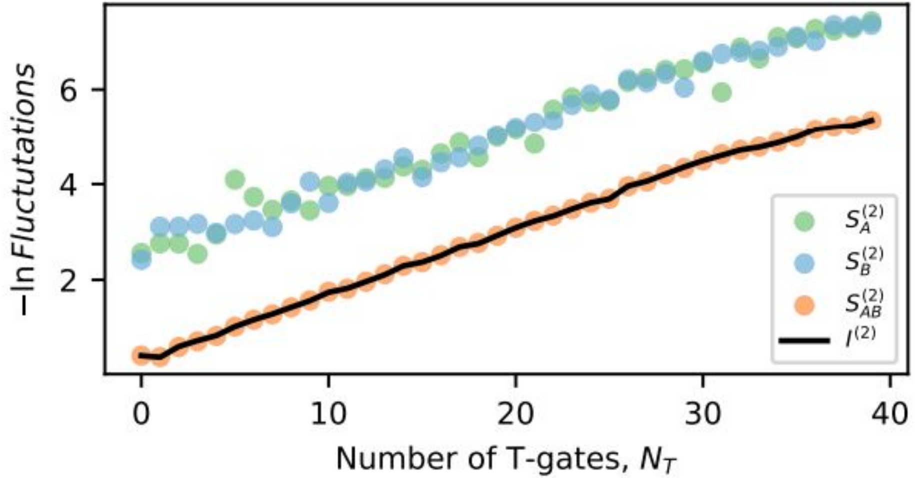


A.A., J.H.,C.K. & E.G., *arXiv:*2408.03831

Effective term in mutual information:

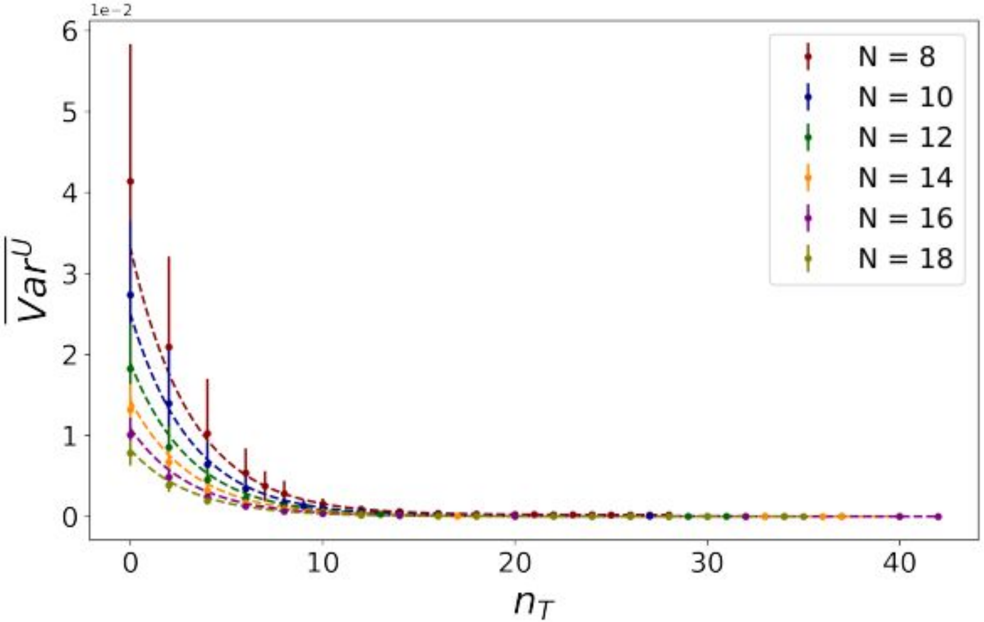
$$I^{(2)} := S_A^{(2)} + S_B^{(2)} - S_{AB}^{(2)}$$

→ |AB| = N/2



A.A., J.H.,C.K. & E.G., *arXiv:*2408.03831

Need to mention:



$$\overline{Var^U} = \frac{0.1}{d^{0.2}} \exp\left[\frac{-n_T}{3.15}\right] + \frac{0.2}{d^{1.25}},$$

Quantum 6, 818 **(2022)**

Analytics:

$$\tilde{S}^{(2)}_{AB} := -\log_2\left(\mathbb{E}\left[\,\mathrm{Tr}\left(\rho^2_{AB}\right)\right]\right),$$
$$\tilde{\delta} := \left(-\log_2\left(\mathbb{E}\left[\,\mathrm{Tr}\left(\rho^2_{AB}\right)^2\right]\right) - (\tilde{S}^{(2)}_{AB})^2\right),$$

⇓

$$-\ln\left(\tilde{\delta}\right) \approx N_T \ln(1/\lambda)$$ ⟹ Linear behaviour!

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Some more intuitive math:

lly. It is important to note that this is not merely a consequence of the fluctuation of the Renyi-2 entropy being **quadratic invariant**.

4-Renyi Entropy is also a quadratic invariant: So if I average over 4-Renyi Entropy I will see this effect as well!

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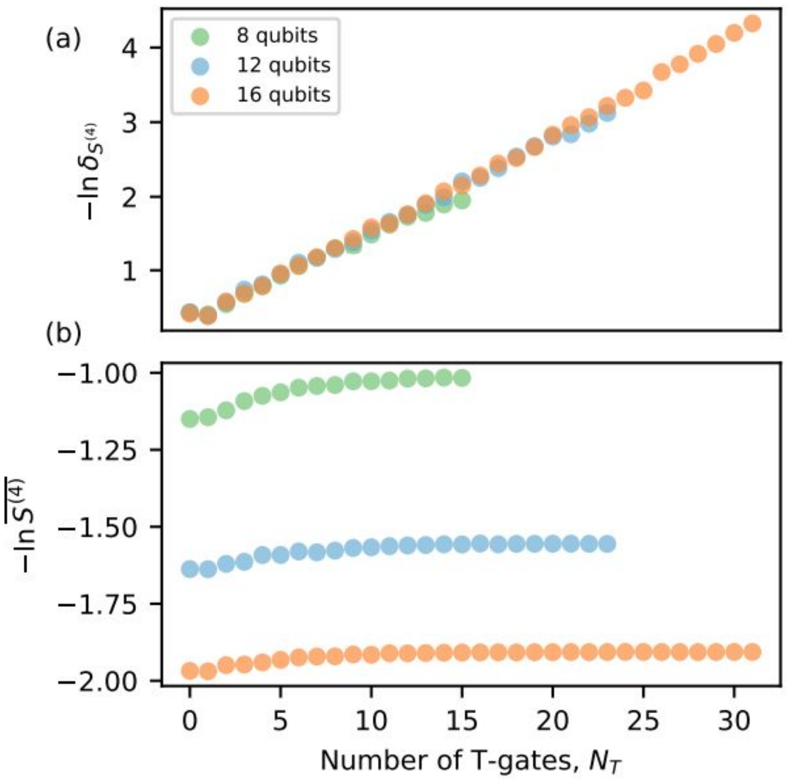
4-Renyi entropy:

$$S_X^{(4)} \equiv -\log_2 tr\rho_X^4$$



The Fluctuations are magical!

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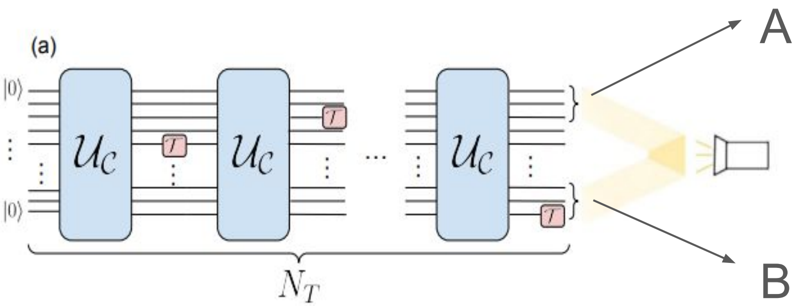
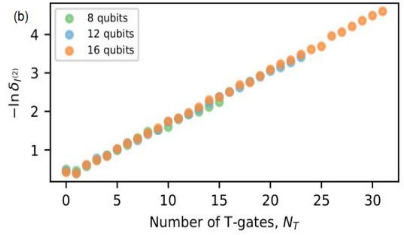
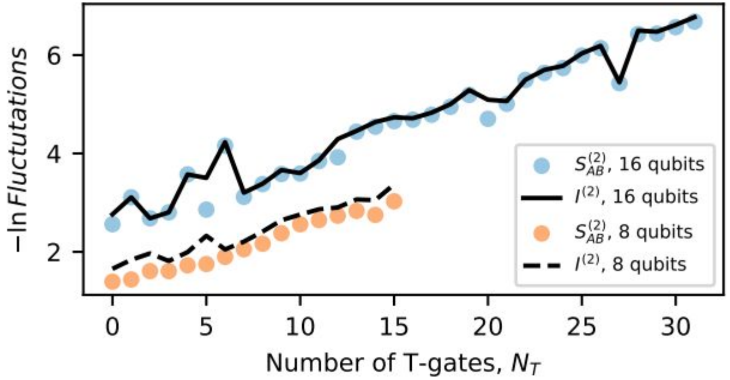
Subsystem Size effect:



When
|A|=|B|=N/4
And |AB|=N/2



What if |A|=|B|=N/8 and
|AB| = N/4 ?



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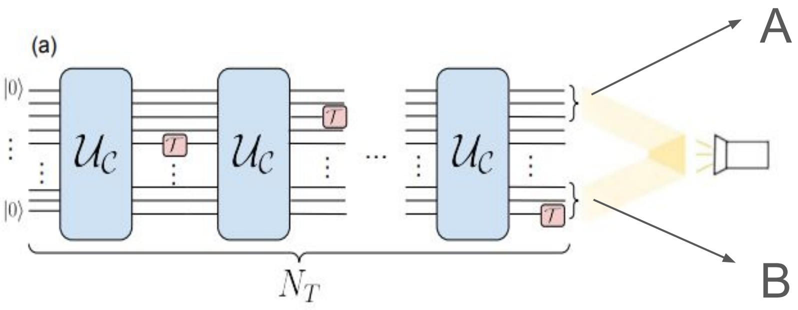
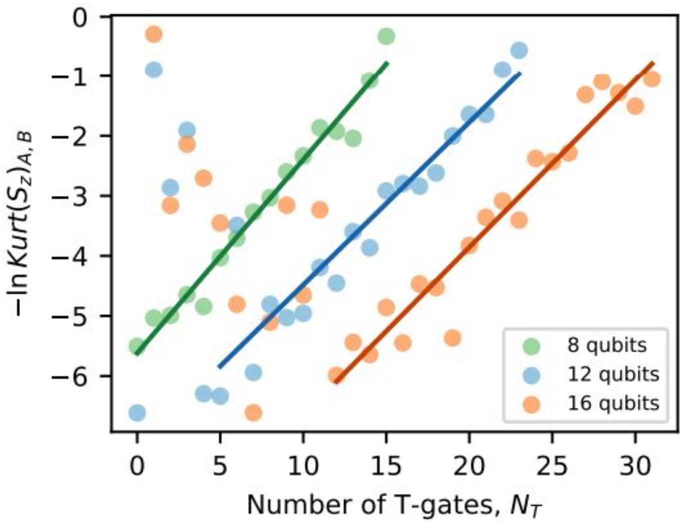
Spin measurement:

$$S_z = \sum_{n\in N_l} s_{n,z} \text{ where } l \in \{A, B\}$$

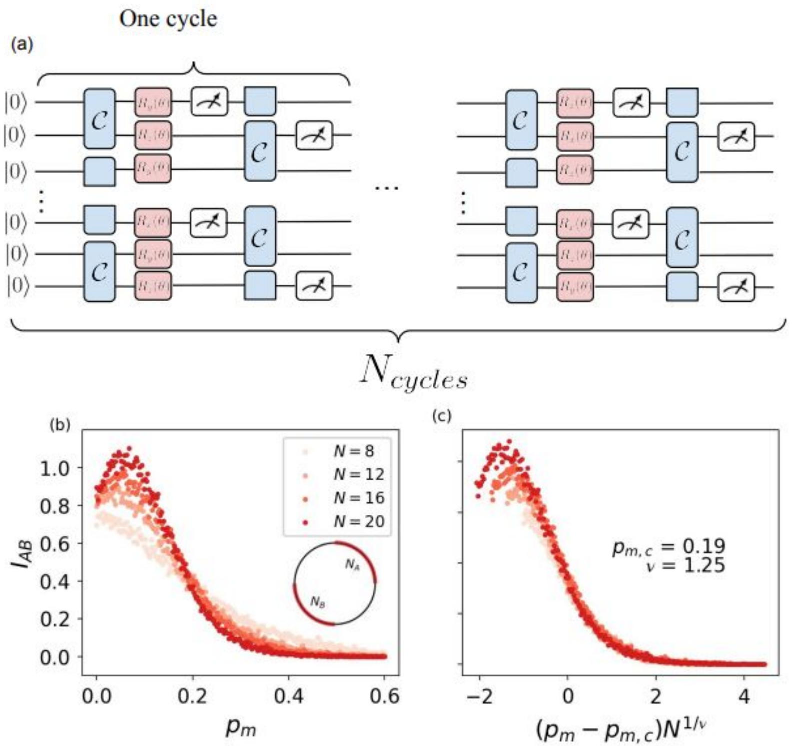




$$Kurt(S_z)_{A,B} := Kurt(\langle S_z\rangle_A) + Kurt(\langle S_z\rangle_B) - Kurt(\langle S_z\rangle_{AB}).$$

A.A., J.H.,C.K. & E.G., *arXiv:*2408.03831

Magic in Measurement-induced Random Circuits:



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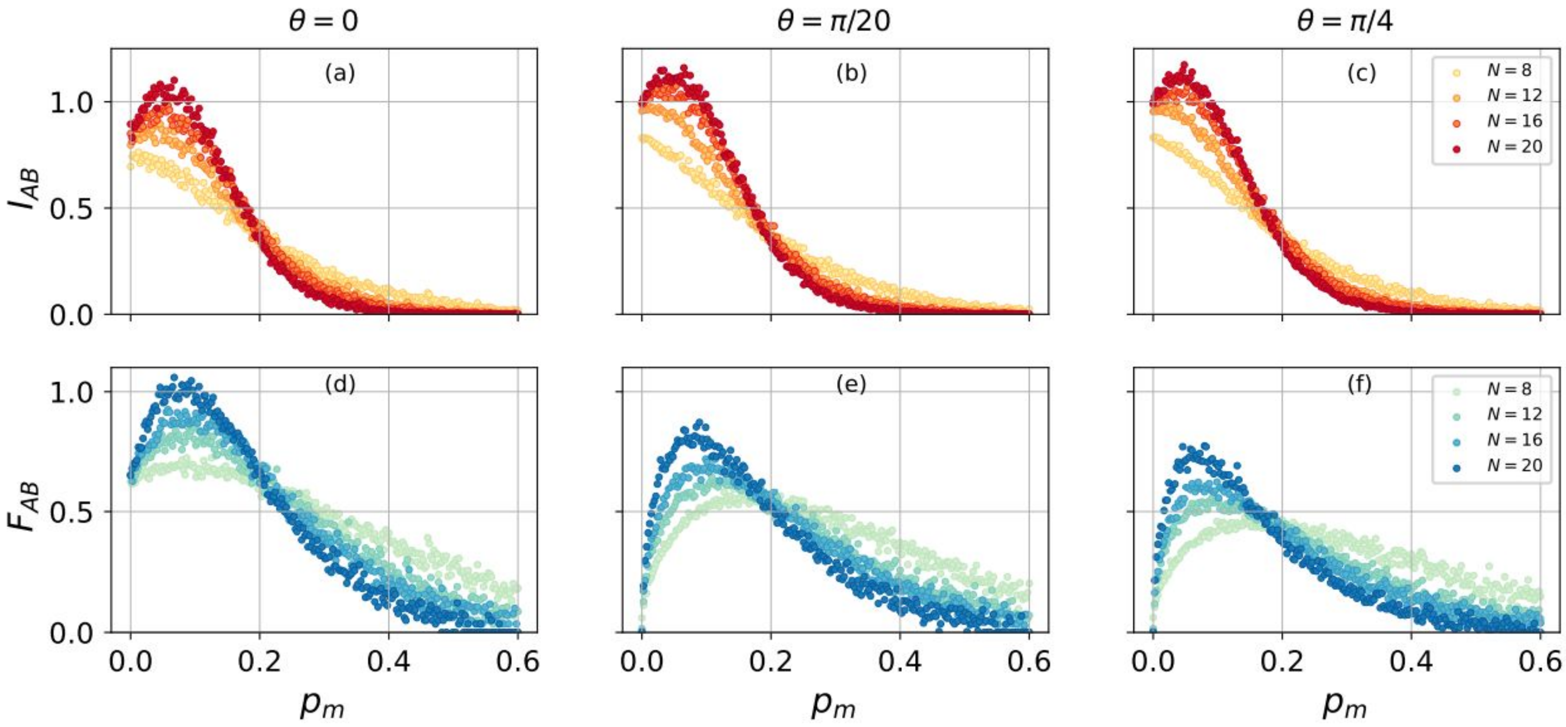
Magic in Measurement-induced Random Circuits:



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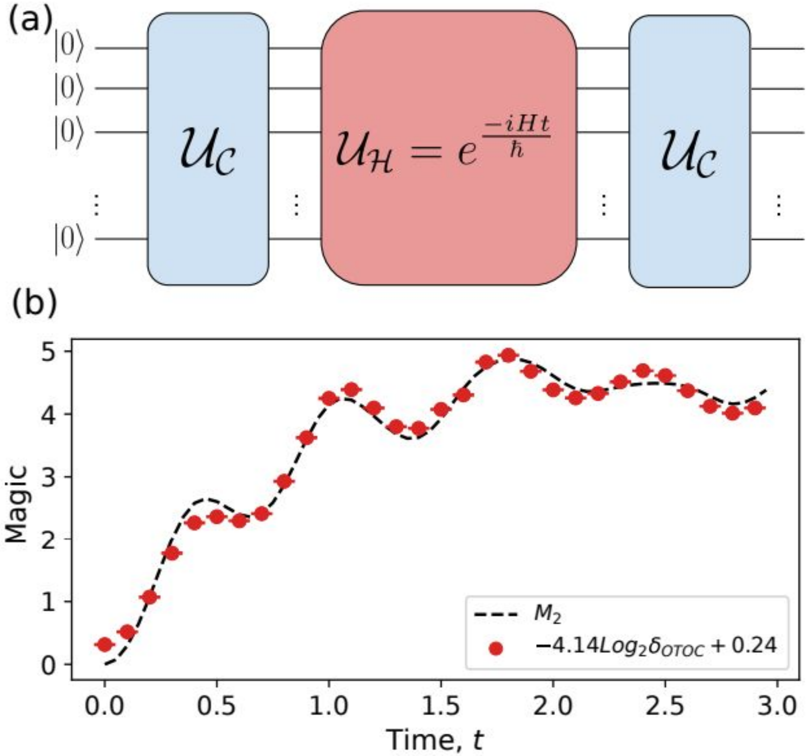
Who cares about magic in random circuits?

Maybe not many people!

What about a generic unitary?



$$H = -J \sum_i Z_i Z_{i+1} - h \sum_i X_i \,.$$

A.A. & E.G. *SciPost Phys.* 16, 043 **(2024)**

Analytics:

$$|V\rangle \equiv \mathbb{I} \otimes V \, |I\rangle \qquad \text{where} \qquad V = \mathcal{U}_{\mathcal{C}} \mathcal{U}_{\mathcal{H}} \mathcal{U}_{\mathcal{C}}$$

$$\mathbb{E}_C \, \delta_{\text{OTOC}}(U) = \left(\frac{d^2}{d^2 - 1} \right)^2 2^{-M_2(|V\rangle)} - \frac{2d^2}{(d^2-1)^2} \, .$$

A.A. & E.G. *SciPost Phys.* 16, 043 **(2024)**

Calculation load:

For Stabilizer Entropies: 4^N Observables to be measured (as the number of Pauli strings)

OTOC fluctuations: Fixed(!) number of OTOC instances (heuristically!)

- Google's experiment had 53 qubits and OTOC instances was 130!
- It is not clear if it is only valid for Random Circuits or it is in general true

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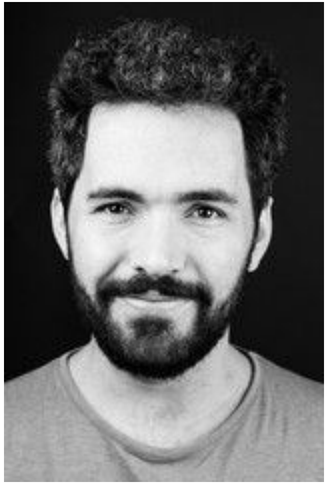
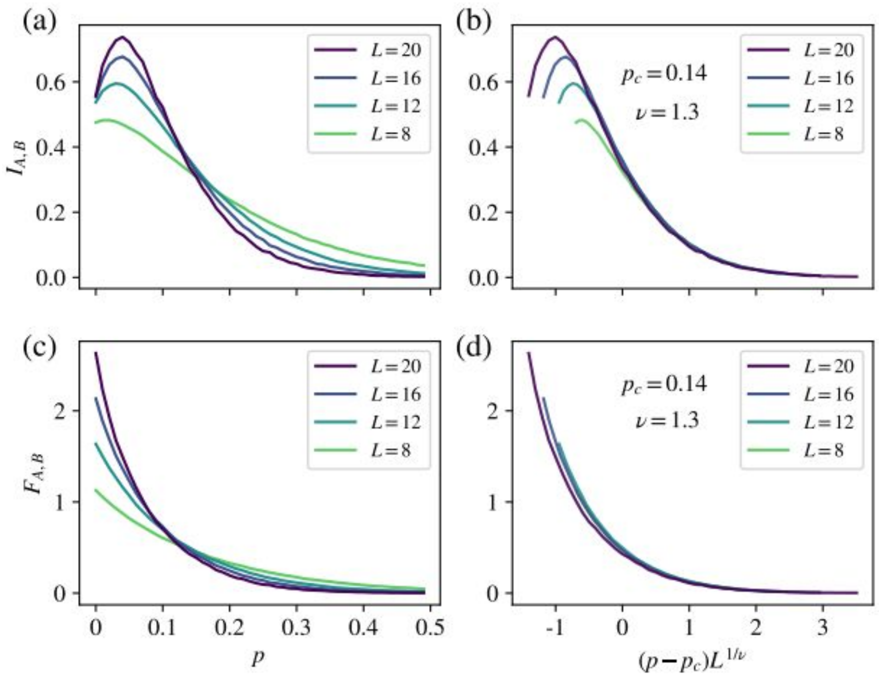
The team

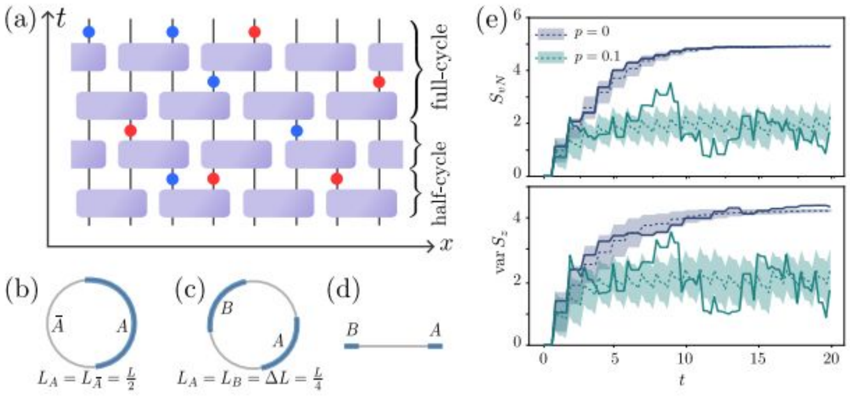


The whole group:

Thank you for your attention!

Spin measurement fluctuations on EPTs:





$$F_{A,B} = \mathrm{var}\, S_{z,A} + \mathrm{var}\, S_{z,B} - \mathrm{var}\, S_{z,A\cup B}$$

Phys. Rev. Lett. 131 020401 (2023)